

Lösung zu den Übungen zur Substitution

Aufgabe	Rechenweg	Ergebnis
$x^4 - 8x^2 + 7 = 0$	$z^2 - 8z + 7 = 0 \Leftrightarrow z_{1,2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - 7} = 4 \pm 3$ $\Leftrightarrow z_1 = 1 \vee z_2 = 7$ $\Leftrightarrow x^2 = 1 \vee x^2 = 7 \xrightarrow{\text{Rücksubstitution}}$ $\Leftrightarrow x = 1 \vee x = -1 \vee x = \sqrt{7} \vee x = -\sqrt{7}$	$x = 1 \vee x = -1 \vee x = \sqrt{7}$ $\vee x = -\sqrt{7}$
$x^4 + 2x^2 - 8 = 0$	$z^2 + 2z - 8 = 0 \Leftrightarrow z_{1,2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-8)} = -1 \pm 3$ $\Leftrightarrow z_1 = 2 \vee z_2 = -4$ $\Leftrightarrow x^2 = 2 \vee x^2 = -4 \text{ (geht nicht)}$ $\Leftrightarrow x = \sqrt{2} \vee x = -\sqrt{2}$	$x = \sqrt{2} \vee x = -\sqrt{2}$
$x^4 + 9x^2 + 20 = 0$	$z^2 + 9z + 20 = 0 \Leftrightarrow z_{1,2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20} = -4,5 \pm 0,5$ $\Leftrightarrow z_1 = -4 \vee z_2 = -5 \Leftrightarrow x^2 = -4 \vee x^2 = -5 \text{ (geht nicht)}$ <p>keine Lösung</p>	keine Lösung
$x^4 - 6x^2 + 8 = 0$	$z^2 - 6z + 8 = 0 \Leftrightarrow z_{1,2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 8} = 3 \pm 1$ $\Leftrightarrow z_1 = 4 \vee z_2 = 2$ $\Leftrightarrow x^2 = 4 \vee x^2 = 2$ $\Leftrightarrow x = 2 \vee x = -2 \vee x = \sqrt{2} \vee x = -\sqrt{2}$	$x = 2 \vee x = -2 \vee x = \sqrt{2}$ $\vee x = -\sqrt{2}$
$x^4 - 2x^2 = 24$	$z^2 - 2z - 24 = 0 \Leftrightarrow z_{1,2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-24)} = 1 \pm 5$ $\Leftrightarrow z_1 = 6 \vee z_2 = -4$ $\Leftrightarrow x^2 = 6 \vee x^2 = -4 \text{ (geht nicht)}$ $\Leftrightarrow x = \sqrt{6} \vee x = -\sqrt{6}$	$x = \sqrt{6} \vee x = -\sqrt{6}$

$x^4 - 10x^2 + 30 = 0$	$z^2 - 10z + 30 = 0 \Leftrightarrow z_{1,2} = -\frac{-10}{2} \pm \sqrt{\left(\frac{-10}{2}\right)^2 - 30} = 5 \pm \sqrt{-5}$ <p>keine Lösung, da die Wurzel aus einer negativen Zahl nicht gezogen werden kann</p>	keine Lösung
$x^4 - 45x^2 + 324 = 0$	$z^2 - 45z + 324 = 0 \Leftrightarrow z_{1,2} = -\frac{-45}{2} \pm \sqrt{\left(\frac{-45}{2}\right)^2 - 324} = 22,5 \pm 13,5$ <p> $\Leftrightarrow z_1 = 36 \vee z_2 = 9$ $\Leftrightarrow x^2 = 36 \vee x^2 = 9$ $\Leftrightarrow x = 6 \vee x = -6 \vee x = 3 \vee x = -3$ </p>	$x = 6 \vee x = -6 \vee x = 3 \vee x = -3$
$x^4 + 4x^2 = -3$	$z^2 + 4z + 3 = 0 \Leftrightarrow z_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 3} = -2 \pm 1$ <p> $\Leftrightarrow z_1 = -1 \vee z_2 = -3$ $\Leftrightarrow x^2 = -1 \vee x^2 = -3$ \Leftrightarrow keine Lösung </p>	keine Lösung
$x^4 - 9x^2 + 14 = 0$	$z^2 - 9z + 14 = 0 \Leftrightarrow z_{1,2} = -\frac{-9}{2} \pm \sqrt{\left(\frac{-9}{2}\right)^2 - 14} = 4,5 \pm 2,5$ <p> $\Leftrightarrow z_1 = 7 \vee z_2 = 2$ $\Leftrightarrow x^2 = 7 \vee x^2 = 2$ $\Leftrightarrow x = \sqrt{7} \vee x = -\sqrt{7} \vee x = \sqrt{2} \vee x = -\sqrt{2}$ </p>	$x = \sqrt{7} \vee x = -\sqrt{7} \vee x = \sqrt{2}$ $\vee x = -\sqrt{2}$
$x^4 - \frac{3}{20}x^2 - \frac{1}{10} = 0$	$z^2 - 2z - 24 = 0 \Leftrightarrow z_{1,2} = -\frac{-\frac{3}{20}}{2} \pm \sqrt{\left(\frac{-\frac{3}{20}}{2}\right)^2 - \left(-\frac{1}{10}\right)} = \frac{3}{40} \pm 0,325 = 0,075 \pm 0,325$ <p> $\Leftrightarrow z_1 = -0,25 \vee z_2 = 0,4$ $\Leftrightarrow x^2 = -0,25$ (geht nicht) $\vee x^2 = 0,4$ $\Leftrightarrow x = \sqrt{0,4} \vee x = -\sqrt{0,4}$ </p>	$x = \sqrt{0,4} \vee x = -\sqrt{0,4}$

Lösen Sie die Gleichung, indem Sie zuerst den Faktor vor x^4 eliminieren!

$2x^4 + 8x^2 - 42 = 0$	$2x^4 + 8x^2 - 42 = 0 /:2$ $\Leftrightarrow x^4 + 4x^2 - 21 = 0$ $z^2 + 4z - 21 = 0 \Leftrightarrow z_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - (-21)} = -2 \pm 5$ $\Leftrightarrow z_1 = 3 \vee z_2 = -7 \Leftrightarrow x^2 = 3 \vee x^2 = -7$ (geht nicht) $\Leftrightarrow x = \sqrt{3} \vee x = -\sqrt{3}$	$x = \sqrt{3} \vee x = -\sqrt{3}$
$4x^4 - 16x^2 - 48 = 0$	$4x^4 - 16x^2 - 48 = 0 /:4$ $\Leftrightarrow x^4 - 4x^2 - 12 = 0$ $z^2 - 4z - 12 = 0 \Leftrightarrow z_{1,2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - (-12)} = 2 \pm 4$ $\Leftrightarrow z_1 = 6 \vee z_2 = -2 \Leftrightarrow x^2 = 6 \vee x^2 = -2$ (geht nicht) $\Leftrightarrow x = \sqrt{6} \vee x = -\sqrt{6}$	$x = \sqrt{6} \vee x = -\sqrt{6}$
$-6x^4 - 12x^2 + 48 = 0$	$-6x^4 - 12x^2 + 48 = 0 /: (-6)$ $x^4 + 2x^2 - 8 = 0$ $z^2 + 2z - 8 = 0 \Leftrightarrow z_{1,2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-8)} = -1 \pm 3$ $\Leftrightarrow z_1 = 2 \vee z_2 = -4 \Leftrightarrow x^2 = 2 \vee x^2 = -4$ (geht nicht) $\Leftrightarrow x = \sqrt{2} \vee x = -\sqrt{2}$	$x = \sqrt{2} \vee x = -\sqrt{2}$
$7x^4 + 21x^2 = -14$	$7x^4 + 21x^2 + 14 = 0 /:7$ $\Leftrightarrow x^4 + 3x^2 + 2 = 0$ $z^2 + 3z + 2 = 0 \Leftrightarrow z_{1,2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 2} = -1,5 \pm 0,5$ $\Leftrightarrow z_1 = -1 \vee z_2 = -2 \Leftrightarrow x^2 = -1 \vee x^2 = -2$ \Leftrightarrow keine Lösung	keine Lösung
$2x^4 - \frac{11}{6}x^2 + \frac{1}{3} = 0$	$2x^4 - \frac{11}{6}x^2 + \frac{1}{3} = 0 /:2$ $x^4 - \frac{11}{12}x^2 + \frac{1}{6} = 0 \Leftrightarrow z^2 - \frac{11}{12}z + \frac{1}{6} = 0 \Leftrightarrow z_{1,2} = -\frac{-\frac{11}{12}}{2} \pm \sqrt{\left(\frac{-\frac{11}{12}}{2}\right)^2 - \frac{1}{6}} = \frac{11}{24} \pm \frac{5}{24}$ $\Leftrightarrow z_1 = \frac{16}{24} = \frac{2}{3} \vee z_2 = \frac{6}{24} = \frac{1}{4} \Leftrightarrow x^2 = \frac{2}{3} \vee x^2 = \frac{1}{4}$ $\Leftrightarrow x = \sqrt{\frac{2}{3}} \vee x = -\sqrt{\frac{2}{3}} \vee x = \frac{1}{4} \vee x = -\frac{1}{4}$	$x = \sqrt{\frac{2}{3}} \vee x = -\sqrt{\frac{2}{3}}$ $\vee x = \frac{1}{4} \vee x = -\frac{1}{4}$