

Lösung zu den Übungen zur Lage zwischen Ebenen in Parameterform und Geraden

Aufgabe 1 und 2 sind mit Matrizen, Aufgabe 3 bis 8 mit dem Gaussverfahren gelöst.

$$1. \quad E: \vec{x} = \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix} + r \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} \quad \text{und} \quad g: \vec{x} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix} + r \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix} \quad / - \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix}$$

$$r \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix} \quad / - t \cdot \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix}$$

$$r \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 4 \end{pmatrix}$$

$$\left| \begin{array}{ccc|c} 4r + 2s + 2t = 8 \\ 3r - 4s + 3t = -2 \\ r + 3s + 2t = 4 \end{array} \right| \quad \text{oder} \quad \left(\begin{array}{ccc|c} 4 & 2 & 2 & 8 \\ 3 & -4 & 3 & -2 \\ 1 & 3 & 2 & 4 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 4 & 2 & 2 & 8 \\ 3 & -4 & 3 & -2 \\ 1 & 3 & 2 & 4 \end{array} \right) \begin{array}{l} I \\ II \\ III \cdot (-3) \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 4 & 2 & 2 & 8 \\ 3 & -4 & 3 & -2 \\ -3 & -9 & -6 & -12 \end{array} \right) \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 4 & 2 & 2 & 8 \\ 3 & -4 & 3 & -2 \\ 0 & -13 & -3 & -14 \end{array} \right) \begin{array}{l} I \cdot 3 \\ II \cdot (-4) \\ III \end{array}$$

$$\left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ -12 & 16 & -12 & 8 \\ 0 & -13 & -3 & -14 \end{array} \right) \begin{array}{l} I \\ I + II \\ III \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 22 & -6 & 32 \\ 0 & -13 & -3 & -14 \end{array} \right) \begin{array}{l} I \\ II \\ III \cdot (-2) \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 22 & -6 & 32 \\ 0 & 26 & 6 & -14 \end{array} \right) \begin{array}{l} I \\ II \\ II + III \end{array}$$

$$\left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 22 & -6 & 32 \\ 0 & 48 & 0 & 60 \end{array} \right) \begin{array}{l} I \\ II \\ III: 48 \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 22 & -6 & 32 \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I \\ II: (-22) \\ III \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & -1 & \frac{6}{22} & -\frac{32}{22} \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I \\ II + III \\ III \end{array}$$

$$\left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 0 & \frac{6}{22} & -\frac{9}{44} \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I \\ II: (\frac{6}{22}) \\ III \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 12 & 6 & 6 & 24 \\ 0 & 0 & 1 & -0,75 \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I: (-6) \\ II \\ III \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} -2 & -1 & -1 & -4 \\ 0 & 0 & 1 & -0,75 \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I + II + II \\ II \\ III \end{array}$$

$$\left(\begin{array}{ccc|c} -2 & 0 & 0 & -3,5 \\ 0 & 0 & 1 & -0,75 \\ 0 & 1 & 0 & 1,25 \end{array} \right) \begin{array}{l} I: (-2) \\ II \\ III \end{array} \Leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1,75 \\ 0 & 1 & 0 & 1,25 \\ 0 & 0 & 1 & -0,75 \end{array} \right) \begin{array}{l} I \\ II \\ III \end{array} \quad \text{d.h. } r = 1,75, s = 1,25, t = -0,75$$

$$g: \vec{x} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} - 0,75 \cdot \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 7,5 \\ 6,25 \\ 3,5 \end{pmatrix}$$

Die Gerade durchstößt die Ebene im Punkt D(7,5/6,25/3,5)!

$$2. \quad E: \vec{x} = \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix} + r \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 9 \\ 3 \end{pmatrix} \quad \text{und} \quad g: \vec{x} = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -6 \\ -2 \end{pmatrix}$$

$$\text{Man erhlt: } \begin{cases} 4r + 2s + 2t = 8 \\ 3r + 9s + 6t = -2 \\ r + 3s + 2t = 4 \end{cases} \quad \text{oder} \quad \begin{pmatrix} 4 & 2 & 2 & 8 \\ 3 & 9 & 6 & -2 \\ 1 & 3 & 2 & 4 \end{pmatrix} \begin{matrix} I \\ II \\ III \cdot (-3) \end{matrix}$$

$$\Leftrightarrow \begin{pmatrix} 4 & 2 & 2 & 8 \\ 3 & 9 & 6 & -2 \\ -3 & -9 & -6 & -12 \end{pmatrix} \begin{matrix} I \\ II \\ III + II \end{matrix} \Leftrightarrow \begin{pmatrix} 4 & 2 & 2 & 8 \\ 3 & 9 & 6 & -2 \\ 0 & 0 & 0 & -14 \end{pmatrix} \begin{matrix} I \\ II \\ III \end{matrix}$$

Die letzte Zeile beinhaltet einen Widerspruch ($0 \neq -14$).

Gerade und die Ebene sind parallel.

$$3. \quad E: \vec{x} = \begin{pmatrix} -4 \\ -5 \\ -10 \end{pmatrix} + r \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{und} \quad g: \vec{x} = \begin{pmatrix} 5 \\ 10 \\ 13 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$$

$$\begin{cases} r + s - t = 9 \\ r + 2s - 4t = 15 \\ r + 3s - 9t = 23 \end{cases} \begin{matrix} I \cdot (-1) \\ II \\ III \end{matrix} \Leftrightarrow \begin{cases} -r - s + t = -9 \\ r + 2s - 4t = 15 \\ r + 3s - 9t = 23 \end{cases} \begin{matrix} I \\ II + I \\ III + I \end{matrix} \Leftrightarrow \begin{cases} s - 3t = 6 \\ 2s - 8t = 14 \end{cases}$$

$$\begin{cases} r + s - t = 9 \\ s - 3t = 6 \\ 2s - 8t = 14 \end{cases} \begin{matrix} I \\ II \cdot (-2) \\ III \end{matrix} \Leftrightarrow \begin{cases} r + s - t = 9 \\ -2s + 6t = -12 \\ 2s - 8t = 14 \end{cases} \begin{matrix} I \\ II \\ III + II \end{matrix}$$

$$\begin{cases} r + s - t = 9 \\ -2s + 6t = -12 \\ -2t = 2 \end{cases} \begin{matrix} I \\ II \\ III \end{matrix} \Leftrightarrow \begin{cases} r + s - t = 9 \\ -2s + 6 \cdot (-1) = -12 \\ t = -1 \end{cases} \begin{matrix} I \\ II \\ III \end{matrix} \Leftrightarrow \begin{cases} r = 5 \\ s = 3 \\ t = -1 \end{matrix} \begin{matrix} I \\ II \\ III \end{matrix}$$

$$\text{Einsetzen in } g: \vec{x} = \begin{pmatrix} 5 \\ 10 \\ 13 \end{pmatrix} + (-1) \cdot \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix}$$

Die Gerade durchstsst die Ebene im Punkt D(4/6/4).

$$4. \quad E: \vec{x} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} + r \cdot \begin{pmatrix} 2 \\ 5 \\ 12 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 2 \\ -9 \end{pmatrix} \quad \text{und} \quad g: \vec{x} = \begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ 10 \\ 8 \end{pmatrix}$$

$$\begin{cases} 2r - s + 4t = 5 \\ 5r + 2s - 10t = 7 \\ 12r - 9s - 8t = 11 \end{cases} \begin{matrix} I \cdot 2 \\ II \\ III \end{matrix} \Leftrightarrow \begin{cases} 4r - 2s + 8t = 10 \\ 5r + 2s - 10t = 7 \\ 12r - 9s - 8t = 11 \end{cases} \begin{matrix} I \\ II + I \\ III \end{matrix} \Leftrightarrow \begin{cases} 9r - 2t = 17 \\ 12r - 9s - 8t = 11 \end{cases}$$

$$\begin{cases} 2r - s + 4t = 5 \\ 5r + 2s - 10t = 7 \\ 12r - 9s - 8t = 11 \end{cases} \begin{matrix} I \cdot (-9) \\ II \\ III \end{matrix} \Leftrightarrow \begin{cases} -18r + 9s - 36t = -45 \\ 5r + 2s - 10t = 7 \\ 12r - 9s - 8t = 11 \end{cases} \begin{matrix} I \\ II + I \\ III \end{matrix} \Leftrightarrow \begin{cases} -6r - 44t = -34 \\ 12r - 9s - 8t = 11 \end{cases}$$

$$\begin{cases} 2r - s + 4t = 5 \\ 9r - 2t = 17 \\ -6r - 44t = -34 \end{cases} \begin{matrix} I \\ II \cdot 2 \\ III \cdot 3 \end{matrix} \Leftrightarrow \begin{cases} 2r - s + 4t = 5 \\ 18r - 4t = 34 \\ -18r - 132t = -102 \end{cases} \begin{matrix} I \\ II \\ III + II \end{matrix} \Leftrightarrow \begin{cases} 2r - s + 4t = 5 \\ 18r - 4t = 34 \\ -136t = -68 \end{cases} \begin{matrix} I \\ II \\ III: (-136) \end{matrix}$$

$$\begin{cases} 2r - s + 4t = 5 \\ 18r - 4 \cdot 0,5 = 34 \\ t = 0,5 \end{cases} \Leftrightarrow \begin{cases} 2 \cdot 2 - s + 4 \cdot 0,5 = 5 \\ r = 2 \\ t = 0,5 \end{cases} \Leftrightarrow \begin{cases} s = 1 \\ r = 2 \\ t = 0,5 \end{cases}$$

$$g: \vec{x} = \begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix} + 0,5 \cdot \begin{pmatrix} -4 \\ 10 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 11 \\ 14 \end{pmatrix}$$

Die Gerade durchstsst die Ebene im Punkt D(2/11/14)!

5. E: $\vec{x} = \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + r \cdot \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + s \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ und g: $\vec{x} = \begin{pmatrix} 8 \\ -3 \\ -4 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -4 \\ 3 \end{pmatrix}$

$$\left| \begin{array}{l} 3r - 4s + 2t = 10 \\ 5r - 3s + 4t = 3 \\ -2r + 5s - 3t = -7 \end{array} \right| \begin{array}{l} I \cdot (-2) \\ II \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} -6r + 8s - 4t = -20 \\ 5r - 3s + 4t = 3 \\ -2r + 5s - 3t = -7 \end{array} \right| \begin{array}{l} I \\ II + I \\ III \end{array} \Leftrightarrow -r + 5s = -17$$

$$\left| \begin{array}{l} 3r - 4s + 2t = 10 \\ 5r - 3s + 4t = 3 \\ -2r + 5s - 3t = -7 \end{array} \right| \begin{array}{l} I \cdot 1,5 \\ II \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 4,5r - 6s + 3t = 15 \\ 5r - 3s + 4t = 3 \\ -2r + 5s - 3t = -7 \end{array} \right| \begin{array}{l} I \\ II \\ III + I \Leftrightarrow 2,5r - s = 8 \end{array}$$

$$\left| \begin{array}{l} 3r - 4s + 2t = 10 \\ -r + 5s = -17 \\ 2,5r - s = 8 \end{array} \right| \begin{array}{l} I \\ II \\ III \cdot 5 \end{array} \Leftrightarrow \left| \begin{array}{l} 3r - 4s + 2t = 10 \\ -r + 5s = -17 \\ 12,5r - 5s = 40 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow \left| \begin{array}{l} 3r - 4s + 2t = 10 \\ -r + 5s = -17 \\ 11,5r = 23 \end{array} \right| \begin{array}{l} I \\ II \\ III: 11,5 \end{array}$$

$$\left| \begin{array}{l} 3r - 4s + 2t = 10 \\ -r + 5s = -17 \\ r = 2 \end{array} \right| \Leftrightarrow \left| \begin{array}{l} 3r - 4s + 2t = 10 \\ -2 + 5s = -17 \\ r = 2 \end{array} \right| \Leftrightarrow \left| \begin{array}{l} 3 \cdot 2 - 4 \cdot (-3) + 2t = 10 \\ s = -3 \\ r = 2 \end{array} \right| \Leftrightarrow \left| \begin{array}{l} t = -4 \\ s = -3 \\ r = 2 \end{array} \right|$$

$$g: \vec{x} = \begin{pmatrix} 8 \\ -3 \\ -4 \end{pmatrix} + (-4) \cdot \begin{pmatrix} -2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 16 \\ 13 \\ -16 \end{pmatrix}$$

Die Gerade schneidet die Ebene im Punkt D(16/13/-16).

6. E: $\vec{x} = \begin{pmatrix} -1 \\ -15 \\ -1 \end{pmatrix} + r \cdot \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 6 \\ 10 \end{pmatrix}$ und g: $\vec{x} = \begin{pmatrix} 3 \\ -7 \\ 11 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ -7 \\ -11 \end{pmatrix}$

$$\left| \begin{array}{l} r + 2s + 3t = 4 \\ 5r + 6s + 7t = 8 \\ 9r + 10s + 11t = 12 \end{array} \right| \begin{array}{l} I \cdot (-5) \\ II \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} -5r - 10s - 15t = -20 \\ 5r + 6s + 7t = 8 \\ 9r + 10s + 11t = 12 \end{array} \right| \begin{array}{l} I \\ II + I \\ III \end{array} \Leftrightarrow -4s - 8t = -12$$

$$\left| \begin{array}{l} r + 2s + 3t = 4 \\ 5r + 6s + 7t = 8 \\ 9r + 10s + 11t = 12 \end{array} \right| \begin{array}{l} I \cdot (-9) \\ II \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} -9r - 18s - 27t = -36 \\ 5r + 6s + 7t = 8 \\ 9r + 10s + 11t = 12 \end{array} \right| \begin{array}{l} I \\ II \\ III + I \Leftrightarrow -8s - 16t = -24 \end{array}$$

$$\left| \begin{array}{l} r + 2s - 2t = -1 \\ -4s - 8t = -12 \\ -8s - 16t = -24 \end{array} \right| \begin{array}{l} I \\ II \cdot (-2) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} r + 2s - 2t = -1 \\ 8s + 16t = 24 \\ -8s - 16t = -24 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow \left| \begin{array}{l} r + 2s - 2t = -1 \\ 8s - 28t = -22 \\ 0 = 0 \end{array} \right| \begin{array}{l} I \\ II \\ III \end{array}$$

III: Es gibt
 ∞ viele
Lösungen.

Die Gerade g liegt in der Ebene E.

7. E: $\vec{x} = \begin{pmatrix} -24 \\ 7 \\ -50 \end{pmatrix} + r \cdot \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + s \cdot \begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$ und g: $\vec{x} = \begin{pmatrix} 2 \\ -5 \\ 9 \end{pmatrix} + t \cdot \begin{pmatrix} 12 \\ -20 \\ 50 \end{pmatrix}$

$$\left| \begin{array}{l} 3r + 6s - 12t = 26 \\ r + 5s + 20t = -12 \\ 5r + 7s - 50t = 59 \end{array} \right| \begin{array}{l} I \\ II \cdot (-3) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 3r + 6s - 12t = 26 \\ -3r - 15s - 60t = 36 \\ 5r + 7s - 50t = 59 \end{array} \right| \begin{array}{l} I \\ II + I \\ III \end{array} \Leftrightarrow -9s - 72t = 62$$

$$\left| \begin{array}{l} 3r + 6s - 12t = 26 \\ r + 5s + 20t = -12 \\ 5r + 7s - 50t = 59 \end{array} \right| \begin{array}{l} I \\ II \cdot (-5) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 3r + 6s - 12t = 26 \\ -5r - 25s - 100t = 60 \\ 5r + 7s - 50t = 59 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow -18s - 150t = 119$$

$$\left| \begin{array}{l} 3r + 6s - 12t = 26 \\ -9s - 72t = 62 \\ -18s - 150t = 119 \end{array} \right| \begin{array}{l} I \\ II \cdot (-2) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 3r + 6s - 12t = 26 \\ 18s + 144t = -124 \\ -18s - 150t = 119 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow -6t = -5$$

$$\left| \begin{array}{l} 3r + 6s - 12t = 26 \\ -12s - 72 \cdot \frac{5}{6} = 62 \\ t = \frac{5}{6} \end{array} \right| \begin{array}{l} I \\ II \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 3r + 6s - 12t = 26 \\ s = -\frac{122}{12} \\ t = \frac{5}{6} \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow \left| \begin{array}{l} r = \frac{352}{9} \\ s = -\frac{122}{12} \\ t = \frac{5}{6} \end{array} \right| \begin{array}{l} I \\ II \\ III \end{array}$$

$$g: \vec{x} = \begin{pmatrix} 2 \\ -5 \\ 9 \end{pmatrix} + \frac{5}{6} \cdot \begin{pmatrix} 12 \\ -20 \\ 50 \end{pmatrix} = \begin{pmatrix} 12 \\ \frac{-65}{3} \\ \frac{152}{3} \end{pmatrix}$$

Die Gerade durchstößt die Ebene im Punkt $D(12/\frac{-65}{3}/\frac{152}{3})$.

8. $E: \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 7 \end{pmatrix} + r \cdot \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ und $g: \vec{x} = \begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix}$

$$\left| \begin{array}{l} 2r + 4s + 4t = -4 \\ 2r + 3s + t = 1 \\ 3r + 5s + 3t = -1 \end{array} \right| \begin{array}{l} I \\ II \cdot (-4) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 2r + 4s + 4t = -4 \\ -8r - 12s - 4t = -4 \\ 3r + 5s + 3t = -1 \end{array} \right| \begin{array}{l} I \\ II + I \\ III \end{array} \Leftrightarrow -6r - 8s = -8$$

$$\left| \begin{array}{l} 2r + 4s + 4t = -4 \\ 2r + 3s + t = 1 \\ 3r + 5s + 3t = -1 \end{array} \right| \begin{array}{l} I \\ II \cdot (-3) \\ III \end{array} \Leftrightarrow \left| \begin{array}{l} 2r + 4s + 4t = -4 \\ -6r - 9s - 3t = -3 \\ 3r + 5s + 3t = -1 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array} \Leftrightarrow -3r - 4s = -4$$

$$\left| \begin{array}{l} 2r + 4s + 4t = -4 \\ -6r - 8s = -8 \\ -3r - 4s = -4 \end{array} \right| \begin{array}{l} I \\ II \\ III \cdot (-2) \end{array} \Leftrightarrow \left| \begin{array}{l} 2r + 4s + 4t = -4 \\ -6r - 8s = -8 \\ 6r + 8s = 8 \end{array} \right| \begin{array}{l} I \\ II \\ III + II \end{array}$$

$$\left| \begin{array}{l} 2r + 4s + 4t = -4 \\ -6r - 8s = -8 \\ 0 = 0 \end{array} \right| \begin{array}{l} I \\ II \\ III \end{array}$$

Die Gerade liegt in der Ebene.