

Lösungen zu den Übungen zur Polynomdivision

1. a. $x^3 - 2x^2 - 5x + 6 : (x + 2) = x^2 - 4x + 3$

$$\begin{array}{r} -) \underline{x^3 + 2x^2} \\ -4x^2 - 5x \\ -) \underline{4x^2 - 8x} \\ 3x + 6 \\ -) \underline{3x + 6} \\ 0 \end{array}$$

b. $x^3 + 8x^2 + 19x + 12 : (x + 3) = x^2 + 5x + 4$

$$\begin{array}{r} -) \underline{x^3 + 3x^2} \\ 5x^2 + 19x \\ -) \underline{5x^2 + 15x} \\ 4x + 12 \\ -) \underline{4x + 12} \\ 0 \end{array}$$

c. $x^3 + 7x^2 - 36 : (x - 2) = x^2 + 9x + 18$

$$\begin{array}{r} -) \underline{x^3 - 2x^2} \\ 9x^2 \\ -) \underline{9x^2 - 18x} \\ 18x - 36 \\ -) \underline{18x - 36} \\ 0 \end{array}$$

d. $x^4 - x^3 - 30x^2 + 72x : (x - 4) = x^3 + 3x^2 - 18x$

$$\begin{array}{r} -) \underline{x^4 - 4x^3} \\ 3x^3 - 30x^2 \\ -) \underline{3x^3 - 12x^2} \\ -18x^2 + 72x \\ -) \underline{-18x^2 + 72x} \\ 0 \end{array}$$

e. $2x^3 + 14x^2 + 14x - 30 : (x - 1) = 2x^2 + 16x + 30$

$$\begin{array}{r} -) \underline{2x^3 - 2x^2} \\ 16x^2 + 14x \\ -) \underline{16x^2 - 16x} \\ 30x - 30 \\ -) \underline{30x - 30} \\ 0 \end{array}$$

2. Löse die Gleichung:

a. $x^3 + 6x^2 - x - 30 = 0$

Nullstelle: $x_0 = 2 : 8 + 24 - 2 - 30 = 32 - 32 = 0$ 😊

$$x^3 + 6x^2 - x - 30 : (x - 2) = x^2 + 8x + 15$$

$$\begin{array}{r} -) \underline{x^3 - 2x^2} \\ 8x^2 - x \\ -) \underline{8x^2 - 16x} \\ 15x - 30 \\ -) \underline{15x - 30} \\ 0 \end{array}$$

$$x^2 + 8x + 15 = 0 \Leftrightarrow x_{1,2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 15} = -4 \pm 1$$

[Alternativ: $x^2 + 8x + 15 : (x + 3) = x + 5$]

$$\begin{array}{r} -) \underline{x^2 + 3x} \\ 5x + 15 \\ -) \underline{5x + 15} \\ 0 \end{array}$$

$$x^3 + 6x^2 - x - 30 = 0$$

$$\Leftrightarrow x = 2 \vee x = -3 \vee x = -5$$

b. $2x^3 + 22x^2 + 40x - 64 = 0 : 2$

$$\Leftrightarrow x^3 + 11x^2 + 20x - 32 = 0$$

Nullstelle: $x_0 = 1 : 1 + 11 + 20 - 32 = 32 - 32 = 0$ 😊

$$x^3 + 11x^2 + 20x - 32 : (x - 1) = x^2 + 12x + 32$$

$$\begin{array}{r} -) \underline{x^3 - x^2} \\ 12x^2 + 20x \\ -) \underline{12x^2 - 12x} \\ 32x - 32 \\ -) \underline{32x - 32} \\ 0 \end{array}$$

$$x^2 + 12x + 32 = 0 \Leftrightarrow x_{1,2} = -\frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 - 32} = -6 \pm 2$$

$$2x^3 + 22x^2 + 40x - 64 = 0$$

$$\Leftrightarrow x = 1 \vee x = -8 \vee x = -4$$

$$c. 2 \cdot (x-3) \cdot (x^3 + x^2 - 14x - 24) = 0$$

$$\Leftrightarrow 2 \cdot (x-3) = 0 \vee x^3 + x^2 - 14x - 24 = 0$$

$$\Leftrightarrow x = 3 \vee x^3 + x^2 - 14x - 24 = 0$$

$$\text{Nullstelle: } x_0 = -2 : -8 + 4 + 28 - 24 = 0 \text{ 😊}$$

$$x^3 + x^2 - 14x - 24 : (x + 2) = x^2 - x - 12$$

$$\begin{array}{r} -) \underline{x^3 + 2x^2} \\ \quad -x^2 - 14x \\ -) \underline{-x^2 - 2x} \\ \quad \quad -12x - 24 \\ -) \underline{-12x - 24} \\ \quad \quad \quad 0 \end{array}$$

$$x^2 - x - 12 = 0 \Leftrightarrow x_{1,2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-12)} = 0,5 \pm 3,5$$

$$2 \cdot (x-3) \cdot (x^3 + x^2 - 14x - 24) = 0$$

$$\Leftrightarrow x = 3 \vee x = -2 \vee x = 4 \vee x = -3$$