Lösungen zu den zusammenfassenden Aufgaben Integralrechnung

1. Finden Sie die zugehörige Stammfunktion!

$f(x) = 2x^2 + 5x$	$F(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2$
$f(x) = 8x^3 - 5x^2 + 3x - 6$	$F(x) = 2x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x$
$f(x) = \frac{2}{3}x^6 - x^5 + \frac{5}{6}x^3 - \frac{9}{16}x^2$	$F(x) = \frac{2}{21}x^7 - \frac{1}{6}x^6 + \frac{5}{24}x^4 - \frac{3}{16}x^3$
$f(x) = 3$	$F(x) = 3x$
$f(x) = 2x^4 \cdot (x^3 + 4)$ $= 2x^7 + 8x^4$	$F(x) = \frac{1}{4}x^8 + \frac{8}{5}x^5$
$f(x) = 4 \cdot e^{3x+4}$	$F(x) = \frac{4}{3} \cdot e^{3x+4}$
$f(x) = e^{-4x+2} + 8 \cdot e^{-6x+1}$	$F(x) = -\frac{1}{4} \cdot e^{-4x+2} - \frac{4}{3} \cdot e^{-6x+1}$
$f(x) = 1 - 8 \cdot e^{-5x+5}$	$F(x) = x + \frac{8}{5} \cdot e^{-5x+5}$
$f(x) = (4x-3)^5$	$F(x) = \frac{1}{24} \cdot (4x-3)^6$

2. Berechnen Sie die Fläche zwischen f(x) und der x-Achse!

$f(x) = 9 - x^2$	$9 - x^2 = 0 \Leftrightarrow x = -3 \vee x = 3$ (f(0) = 9, also f(x) > 0 in [-3;3]) $A = \int_{-3}^{3}(9 - x^2)dx = \left[9x - \frac{1}{3}x^3\right]_{-3}^{3} = 18 - (-18) = \mathbf{36}$
$f(x) = x^2 - 5x - 24$	$x^2 - 5x - 24 = 0 \Leftrightarrow x = -3 \vee x = 8$ (f(0) = -24, also f(x) < 0 in [-3;8]) $A = \left\|\int_{-3}^{8}(x^2 - 5x - 24)dx\right\| = \left\|\left[\frac{1}{3}x^3 - 2{,}5x^2 - 24x\right]_{-3}^{8}\right\|$ $= \|-181{,}\overline{3} - 40{,}5\| = \|-221{,}8\overline{3}\| = \mathbf{221{,}8\overline{3}}$
$f(x) = x^3 + 3x^2 - 10x - 24$	$x^3 + 3x^2 - 10x - 24 = 0 \Leftrightarrow x = -4 \vee x = -2 \vee x = 3$ $A = \left\|\int_{-4}^{-2}(x^3 + 3x^2 - 10x - 24)dx\right\| + \left\|\int_{-2}^{3}(x^3 + 3x^2 - 10x - 24)dx\right\|$ $= \left\|\left[\frac{1}{4}x^4 + x^3 - 5x^2 - 24x\right]_{-4}^{-2}\right\| + \left\|\left[\frac{1}{4}x^4 + x^3 - 5x^2 - 24x\right]_{-2}^{3}\right\|$ $= 8 + \|-93{,}75\| = \mathbf{101{,}75}$

3. Berechnen Sie die Fläche zwischen den beiden Funktionen!

$f(x) = -8x + 12$ $g(x) = 4x^2$	$-8x + 12 = 4x^2 \Leftrightarrow 4x^2 + 8x - 12 = 0 \Leftrightarrow x = -3 \vee x = 1$ (f(0) = 12, g(0) = 0 => f(x) > g(x) in [−3;1]) $A = \int_{-3}^{1}(-4x^2 - 8x + 12)dx = \left[-\frac{4}{3}x^3 - 4x^2 + 12x\right]_{-3}^{1}$ $= \frac{20}{3} - (-36) = \mathbf{\frac{128}{3} = 42,\overline{6}}$
$f(x) = x^2 - x^4$ $g(x) = x^2 - 1$	$x^2 - x^4 = x^2 - 1 \Leftrightarrow x^4 = 1 \Leftrightarrow x = -1 \vee x = 1$ (f(0) = 0, g(0) = −1 => f(x) > g(x) in [−1;1]) $A = \int_{-1}^{1}(-x^4 + 1)dx = \left[-\frac{1}{5}x^5 + x\right]_{-1}^{1} = \frac{4}{5} - (-\frac{4}{5})$ $= \mathbf{\frac{8}{5} = 1,6}$

4. Berechnen Sie den Mittelwert der Funktion im angegebenen Intervall I!

$f(x) = -x^2+16$ $I = [1;3]$	$\overline{m} = \frac{1}{3-1} \cdot \int_{1}^{3}(-x^2 + 16)dx = \frac{1}{2} \cdot \left[-\frac{1}{3}x^3 + 16x\right]_{1}^{3} = \frac{1}{2} \cdot \frac{70}{3}$ $= \mathbf{\frac{70}{6} = 11,\overline{6}}$
$f(x) = \frac{1}{4}x^4 - 2x^3$ $I = [-1;5]$	$\overline{m} = \frac{1}{5-(-1)} \cdot \int_{-1}^{5}\left(\frac{1}{4}x^4 - 2x^3\right)dx = \frac{1}{6} \cdot \left[\frac{1}{20}x^5 - \frac{1}{2}x^4\right]_{-1}^{5}$ $= -\mathbf{\frac{155,7}{6} = -25,95}$
$f(x) = 6x^3$ $I = [-3;3]$	$\overline{m} = \frac{1}{6} \cdot \int_{-3}^{3} 6x^3 dx = \frac{1}{6} \cdot \left[\frac{3}{2}x^4\right]_{-3}^{3} = \frac{1}{6} \cdot \left(\frac{243}{2} - \frac{243}{2}\right) = \mathbf{0}$